# Animating Mathematics with TI-Nspire Computer Software

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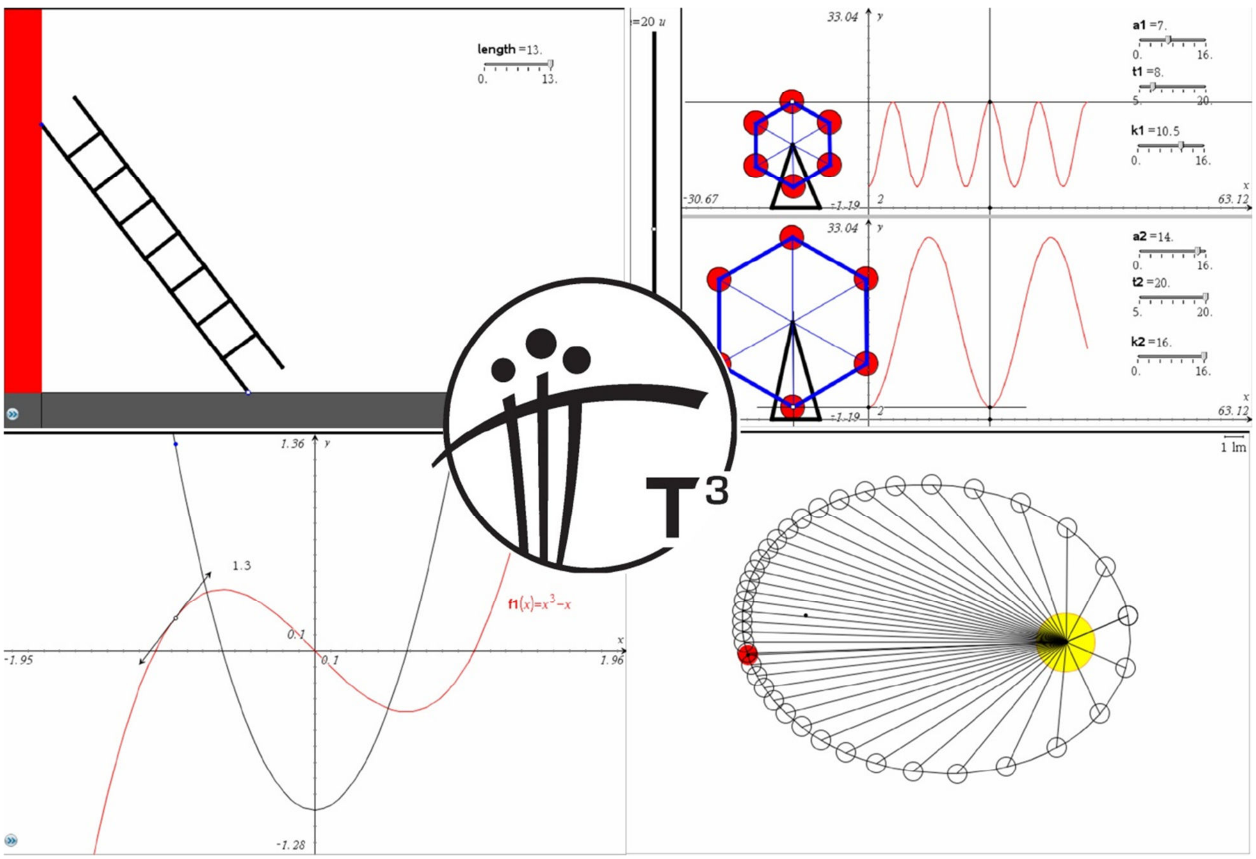


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# Graphs of Derivatives

*"Another thing it allows us to do is reorder the curriculum. Traditionally it's been by how difficult it is to calculate, but now we can reorder it by how difficult it is to understand the concepts, however hard the calculating. So calculus has traditionally been taught very late. Why is this? Well, it's damn hard doing the calculations, that's the problem. But actually many of the concepts are amenable to a much younger age group."*

*Conrad Wolfram*
*Teaching kids real math with computers*
http://www.ted.com/talks/lang/eng/conrad_wolfram_teaching_kids_real_math_with_computers.html

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| 1. Create a new document with a Graphs page. | ctrl N 2 |
| 2. Enter the function $\mathbf{f1}(x) = x^3 - x$. | |
| 3. Grab the axis and zoom into the window so the x-axis goes from approximately -1.7 to 1.7. | ctrl on axis and move. |
| 4. Place a tangent line on the graph of **f1**. | menu 7 7 |
| 5. Measure the slope of the tangent line. | menu 8 3 |
| 6. Transfer the measurement of the slope to the y-axis. | menu A 8 |
| 7. Construct a line perpendicular to the y-axis through this point. You may have to move the tangent line to get this point in the window. | menu A 1 |
| 8. Construct a line perpendicular to the x-axis through the point of tangency. | menu A 1 |
| 9. Plot a point of intersection between these two lines. | menu 7 3 |
| 10. Hide everything that doesn't need to be seen by the student. | menu 1 3 |
| 11. Change the attribute of the point of intersection to large (8/9). | ctrl menu 3 |
| 12. Change the attribute of the point of tangency to empty large (9/9) and set the animation speed to 1. | ctrl menu 3 |
| 13. Copy this page as needed to new problems. | |

The "Graphs of Derivatives" document has what we created for a specially selected set of functions.

# Take a Ride on the Ferris Wheel
# Experiencing Periodic Functions

*A Ferris wheel is 10 meters in diameter and is boarded from a platform that is 2.5 meters above the ground. Assume the 6 o'clock position on the Ferris wheel is level with the loading platform. The wheel completes one full revolution every 12 minutes. Draw a graph of the function* $h(t)$*, the height in meters of the rider on the Ferris wheel at a time* $t$ *in minutes. Label the amplitude, period, and midline.*

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| 14. Create a new document with a Graphs page. | ctrl N 2 |
| 15. Hide the entry line. | ctrl G |
| 16. Right-Click Chevron and choose hide chevron. | ctrl menu 3 |
| 17. Define the window.<br>We will place the Ferris wheel in quadrant II.<br>We will set the window easily by moving and stretching the axes. | ctrl ✍ in space and move.<br>ctrl ✍ on axis and move. |
| 18. Plot the center of the Ferris wheel (-12.5,12.5) | menu 7 1 ( − 1 2 . 5 enter 1 2 . 5 enter |
| 19. Show the coordinates of the center. | ctrl menu 7 |
| 20. Next we will create a slider to represent our time variable. Place a point on the y-axis and extend the segment to right to create a horizontal segment. Holding down shift will ensure that the segment is horizontal. Next create a second segment on top of the first segment using the point on the y-axis and any second point on the original segment. | menu 7 5<br>Click enter when first point is on y-axis. Holding ⇧shift, place second point and click enter. |
| 21. Measure the new segment. This length will be our time variable. | menu 8 1 |
| 22. Next we will enter the text for the parametric expressions for the location of our rider. Create two text boxes with the following expression inside.<br>$-10\cos\left(\left(\frac{2*pi}{12}\right)*(t)\right)+y\_center$ and $10\sin\left(\left(\frac{2*pi}{12}\right)*(-t)\right)+x\_center$<br>The first expression will be the height of our rider and the second equation will be the horizontal location of our rider with respect to the center we have chosen. | menu 1 6<br>ctrl ␣ for _ |

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| 23. Calculate each of the above text boxes using the length of the segment for $t$, the x-coordinate of our center for $x\_center$ and the y-coordinate of our center for $y\_center$. | menu 1 8 |
| 24. Transfer the calculation for the first equation to the y-axis. Transfer the calculation for the second equation to the x-axis. | menu A 8 |
| 25. Construct lines through these point perpendicular to their respective axis. | menu A 1 |
| 26. Plot a point at the intersection of these two perpendicular lines. | menu 7 3 |
| 27. Draw a circle for the center of the Ferris wheel to this intersection point. | menu 9 1 |
| 28. Hide stuff that doesn't need to be seen by student. | menu 1 3 |
| 29. Change the attributes of the point on our "slider" segment. Make the point empty large (9/9) and change the animation speed from 0 to 1. | ctrl menu 3 |
| 30. Change the attributes of the point for the rider to empty circle (2/9). | ctrl menu 3 |
| 31. Create places for other riders by constructing a regular polygon from the Ferris wheel center to the point for our rider and choosing the appropriate number of sides. | menu 9 5 |
| 32. Create seats for each rider using the compass tool. I generally let the radius of my circle be either the x-scale or y-scale depending on the problem. | menu A 7 |
| 33. Label point on slider as "time = ". | ctrl menu 2 |
| 34. Attach "time" measurement to the right of this label. | ctrl menu 8 |
| 35. Another useful tool to use in this construction is the locus. | menu A 6 |

The periodic function document has several problems that continue to build on this animation.
Problem 1 is the polished version of the above.
Problem 2 allows for the amplitude, period, and midline of two Ferris wheels to be studied on the same screen.
Problem 3 studies the "rolling" Ferris wheel i.e. cycloids.
Problem 4 shows the periodic motion of a double spring system.
Problem 5 shows the spring system moving along the axis of time.
Problem 6 allows for the period of the spring system to be modified.

# Constructions, Conics, and Kepler's Law

## Parabola

*A parabola is the locus of all points in the plane equidistant from a given line and a given point not on the line.*

1. Draw a line (Directrix).
2. Plot a point not on the line (Focus).
3. Plot a point on the line.
4. Construct the perpendicular bisector between this point and the Focus.
5. Construct a line perpendicular to the Directrix through the point plotted on the directrix.
6. Plot the intersection of this perpendicular line and the perpendicular bisector.
7. Construct the locus of this intersection constrained by the point plotted on the line.

## Ellipse

*An ellipse is the locus of all points of the plane where the sum of the distances to two fixed points is a constant.*

1. Plot two points for the Foci (F1 and F2).
2. Draw a circle with one focus (F1) as the center and the other focus (F2) located on the interior the circle.
3. The radius of the circle represents the constant sum.
4. Draw a line between F1 and a point on the circle.
5. Construct the perpendicular bisector between F2 and the point on the circle.
6. Find the intersection of the line through F1 and the perpendicular bisector.
7. Construct the locus of the intersection point constrained by the point on the circle.

## Hyperbola

*A hyperbola is the locus of all points of the plane where the difference of the distances to two fixed points is a constant.*

1. Copy the ellipse page to a new page.
2. Move F2 outside of the radius of the circle.

The “Keplers Law” document is built on this construction.
The “Bending Directrix” document uses geometric inversions to “bend” the directrix of the parabola into a circle.

# Related Rates and the Falling Ladder
# Data from a Virtual Laboratory

*The bottom of a 10-foot ladder is going away from the wall at dx/dt = 2 feet per second. How fast is the top going down the wall? Draw the right triangle to find dy/dt when the height y is (a) 6 feet (b) 5 feet (c) zero.*

*The top of the 10-foot ladder can go faster than light? At what height y does dy/dt = -c?*

*G. Strang, Calculus, Wellesley Cambridge Press, Wellesley, MA, 1991, p. 164.*

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| 36. Create a new document with a Graphs page. | ctrl N 2 |
| 37. Hide the entry line. | ctrl G |
| 38. Right-Click Chevron and choose hide chevron. | ctrl menu 3 |
| 39. Define the window. This will all take place in the first quadrant. We will set the window easily by moving and stretching the axes. | ctrl in space and move.<br>ctrl on axis and move. |
| 40. Construct a circle centered at the origin with a radius of 10 using the compass tool. | menu A 7 |
| 41. Plot the intersection point between the circle and the x-axis. | menu 7 1 |
| 42. Draw a segment along the x-axis from the intersection point to the origin. | menu 7 5 |
| 43. Plot a point on this segment. This point is the base of our ladder. | menu 7 1 |
| 44. Draw a circle centered at this base point with a radius of 10 using the compass tool. | menu A 7 |
| 45. Plot the point of intersection between this new circle and the y-axis. This point is the other end of our ladder. | menu 7 1 |
| 46. Connect the two ends of our ladder using a segment. | menu 7 5 |
| 47. Hide everything the student doesn't need to see. | menu 1 3 |
| 48. Change the attributes of the segment so that the line weight is thick (3/3). | ctrl menu 3 |

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| 49. Change the attributes of the point on the y-axis to be empty circle(2/9). | ctrl menu 3 |
| 50. Change the attributes of the point on the x-axis to be empty large(9/9). | ctrl menu 3 |

The “Falling Ladder Document has three problems.
Problem 1 is the page we just created.
Problem 2 is the same page we created but with the values of the base and height linked to variable and collected in a list and spreadsheet page. The scatterplot of the height and base trace its path. Page 2.3 shows a scatterplot of the heights vs. bases.
Problem 3 is a prettier version of the ladder problem where the length of ladder can be changed with a slider.